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Classical time-symmetric electrodynamics

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Abstract. A brief review of the classical aspects of the absorber theory of radiation is presented. Difficulties in the arguments used by earlier authors are discussed. The divergences which arise from the use of time-symmetric electrodynamics are pointed out. It is shown that the earlier difficulties can be removed by attributing differing signal velocities to advanced and retarded interactions. This difference in signal velocities is interpreted as arising from the extended, shell-like structure of charged particles. This leads to a new calculation of the absorber response. Absorption due to the time-symmetry normalisation factor is described. It is concluded that retarded radiation is approximately consistent in the Einstein–de Sitter model, whereas in the closed Friedman model it is likely that retarded radiation is dominant during expansion, and advanced radiation during contraction. The theory predicts that advanced radiation exists in small amounts and can be detected experimentally.

1. Introduction

The scalar wave equation in flat space

$$\Box^2\psi(\boldsymbol{r}, t) = 4\pi\rho(\boldsymbol{r}, t) \tag{1.1}$$

(where $\Box^2 = \nabla^2 - (\partial^2/\partial t^2)$ is the wave operator, $\psi(\boldsymbol{r}, t)$ is the wave amplitude at the space–time point $(\boldsymbol{r}, t)$ and $\rho(\boldsymbol{r}, t)$ is the source density at $(\boldsymbol{r}, t)$) has two types of solution (Davies 1974): $\psi_{\mathrm{r}} = \psi(\boldsymbol{r}, t^+)$ and $\psi_{\mathrm{a}} = \psi(\boldsymbol{r}, t^-)$ where

$$\begin{aligned} &\psi(\boldsymbol{r}, t) = \int_{\mathbb{R}^3} (\rho(\boldsymbol{r}', t')/R)\, \mathrm{d}^3 r' \\ &R = |\boldsymbol{r} - \boldsymbol{r}'| \qquad t^{\pm} = t \pm R. \end{aligned} \tag{1.2}$$

These are known (with obvious notation) as the retarded and the advanced solutions, and represent waves propagating into the future and the past respectively. In curved space, with metric tensor $g^{\mu\nu}$, the corresponding retarded and advanced solutions for the scalar and vector wave equations

$$g^{\mu\nu}\psi_{;\mu} = 0 \tag{1.3}$$

$$g^{\mu\nu}A^{\sigma}{}_{;\mu\nu} + R^{\sigma\alpha}A_{\alpha} = 0, \tag{1.4}$$

where $R^{\sigma\alpha}$ is the Ricci tensor, have been studied by De Witt and Brehme (1960).

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Mathematically, any normalised combination

$$\psi = \theta\psi_a + (1-\theta)\psi_r \tag{1.5}$$

is also a solution, and the choice of the correct solution depends on the boundary conditions imposed. But, physically, boundary conditions cannot be imposed at will, and the existence of solutions with an advanced component would contradict our usual ideas of causality. The advanced solutions are rejected for this reason. However, it was pointed out by Dirac (1938) that this semi-empirical rejection may not be well founded, because a covariant derivation of the radiative damping force leads to the expression

$$\tfrac{2}{3}e^2(\dddot{z}^{\mu} + \dot{z}^{\mu}\ddot{z}^2) = e\dot{z}\tfrac{1}{2}(F^{\mu}_{r\nu} - F^{\mu}_{a\nu}) \tag{1.6}$$

(where e is the charge, z^{μ} the world line, $F^{\mu}_{r\nu}$, $F^{\mu}_{a\nu}$ the retarded and advanced fields of the particle and dots denote differentiation with respect to the parameter), apparently necessitating the use of advanced solutions.

Thus the problem is to specify the physical nature of the boundary conditions which give rise to the retarded solutions of experience, starting from solutions of the form (1.5). This problem is tackled in the absorber theory of radiation, initiated by Wheeler and Feynman (1945, 1949), and developed by Hogarth (1962), Hoyle and Narlikar (1964, 1969, 1971, 1972), Davies (1970, 1971, 1972a) and others. This direct particle interaction theory uses the Schwarzschild–Tetrode–Fokker action

$$J = -\sum_i m_i \int (\dot{z}^{\mu}_{(i)}\dot{z}_{(i)\mu})^{1/2}\,\mathrm{d}\tau_i - \tfrac{1}{2}\sum_{i\neq j} e_i e_j \int \dot{z}^{\mu}_{(i)}\delta[(z^{\mu}_{(i)} - z^{\mu}_{(j)})(z_{(i)\mu} - z_{(j)\mu})]\dot{z}_{(j)\mu}\,\mathrm{d}\tau_i\,\mathrm{d}\tau_j \tag{1.7}$$

(where $z_i = z_i(\tau_i)$ is the world line of the ith particle with charge e_i and mass m_i, τ_i is the ith particle proper time and δ is the Dirac delta function).

In analogy with field theory, the last term of (1.7) can be used to define the four-potential

$$A_{\mu}(x) = \sum_j e_j \int \delta[(x^{\mu} - z^{\mu}_{(j)})(x_{\mu} - z_{(j)\mu})]\dot{z}_{(j)\mu}\,\mathrm{d}\tau_j. \tag{1.8}$$

This four-potential satisfies the Lorentz condition $A^{\mu}{}_{,\mu} = 0$ and the electromagnetic wave equation

$$\Box^2 A_{\mu}(x) = 4\pi \sum_j e_j \int \delta(x - z_{(j)\mu})\dot{z}_{(j)\mu}\,\mathrm{d}\tau_j \overset{\text{def}}{=} 4\pi j_{\mu}(x). \tag{1.9}$$

For consistency with (1.7), the unique solution of (1.9) is

$$A^{\mu} = \tfrac{1}{2}(A^{\mu}_r + A^{\mu}_a) \tag{1.10}$$

where $A^{\mu}_r = A^{\mu}_*(t^-)$, $A^{\mu}_a = A^{\mu}_*(t^+)$ and

$$A^{\mu}_*(t') = \int (j^{\mu}(r', t')/R)\,\mathrm{d}^3 r'. \tag{1.11}$$

If we re-introduce fields $F^{\mu}{}_{\nu}$, defined as usual by

$$\begin{aligned} F^{\mu}_{r\nu} &= A^{\mu}_{r,\nu} - A^{\nu}_{r,\mu} \\ F^{\mu}_{a\nu} &= A^{\mu}_{a,\nu} - A^{\nu}_{a,\mu}, \end{aligned} \tag{1.12}$$

then the electromagnetic fields obtained in this theory are given by

$$F^{\mu\nu} = \tfrac{1}{2}F_{\rm r}^{\mu\nu} + \tfrac{1}{2}F_{\rm a}^{\mu\nu}. \tag{1.13}$$

Hoyle and Narlikar (1964) have generalised this procedure to curved space, using the appropriate Green functions in the action (1.7). Equation (1.13) can also be obtained from the physical argument that Maxwell's equations, being time symmetric, should not by themselves impose an arrow of time on the solutions.

The usual retarded fields are now obtained by adding to (1.13) the radiative damping term

$$F_{\rm rad} = \tfrac{1}{2}F_{\rm r} - \tfrac{1}{2}F_{\rm a}. \tag{1.14}$$

(Tensor indices will be dropped henceforth as they are not necessary.) For a given particle, the term (1.14) was interpreted by Wheeler and Feynman (1945; WF) as the response of an ideal absorber to the elementary time-symmetric field of the charged particle. Thus the existence of purely retarded radiation gives a condition on the real universe, namely it should be totally absorbing (example—static, Euclidean universe). Hoyle and Narlikar (1964; HN) gave the differing condition that the universe should be opaque along the future null cone and transparent along the past null cone (example—steady-state model). In the following, shortcomings in the arguments of both WF and HN are pointed out, and it is shown that a more plausible condition is that the universe should be opaque along the past null cone, but not totally absorbing.

One other possibility that needs to be mentioned here is that the common presumption that purely retarded radiation exists in the universe may simply not be true. Attempts to confirm experimentally the presence or absence of advanced radiation have been made, and are continuing (Partridge 1973, Heron and Pegg 1974, Pegg 1975a, Davies 1975). However, it is hardly possible to interpret the results of these experiments in the absence of a satisfactory theory. The present attempt to construct a satisfactory theory arises from an attempt to remove the difficulties faced by the earlier theories of WF and HN. Hence these difficulties will be considered first.

2. Theoretical difficulties

2.1. Self-consistency

The fundamental problem of time-symmetric electrodynamics (TSE) is to reconcile the following two facts; the action principle (1.7) permits only time-symmetric fields, while the usual fields of experience are, at least approximately, retarded. In WF it is proposed that this problem could be looked upon either from a general point of view or as a matter of explicit calculation. Only the general point of view is used in HN, and we consider this first.

The total field acting on a charged particle i is given, according to (1.13), by

$$F_{\rm tot} = \tfrac{1}{2}\sum_{j\neq i} F_{\rm r}^{j} + \tfrac{1}{2}\sum_{j\neq i} F_{\rm a}^{j} \tag{2.1}$$

where the field of the jth particle is indexed by j, and the summation ranges over all other particles j in the universe. On the other hand, to account for the observed fully retarded fields and radiation damping, the total field should be of the form

$$F_{\rm tot} = \sum_{j\neq i} F_{\rm r}^{j} + \tfrac{1}{2}(F_{\rm r}^{i} - F_{\rm a}^{i}). \tag{2.2}$$

The problem of self-consistency is, then, to decide the circumstances under which (2.1) is consistent with (2.2). A necessary and sufficient condition for (2.1) to be consistent with (2.2) is, clearly,

$$\sum_i (F^i_{\mathrm{r}} - F^i_{\mathrm{a}}) = 0. \tag{2.3}$$

The general point of view, mentioned above, aims to show that (2.3) is valid under various plausible physical conditions. However, by subtracting (2.3) from (2.1)

$$F_{\mathrm{tot}} = \sum_{j \neq i} F^j_{\mathrm{a}} + \tfrac{1}{2}(F^i_{\mathrm{a}} - F^i_{\mathrm{r}}) \tag{2.4}$$

and, conversely, (2.1) and (2.4) together imply (2.3). It follows that (2.2) is consistent with (2.1) if and only if (2.4) is, i.e., retarded fields with radiative damping are consistent if and only if advanced fields with radiative anti-damping are simultaneously consistent.

Two explanations have been offered for this apparently paradoxical situation. According to WF, the particles on the past null cone of i may be assumed to be in a state of random motion, i.e., their motion is uncorrelated with the motion of particle i. Hence, $\Sigma_{j\neq i} F^j_{\mathrm{r}}$ is small compared with the radiative damping term. On the other hand, the fields F^j_{a} are highly correlated with the motion of i, and it may be imagined that $\Sigma_{j\neq i} F^j_{\mathrm{a}} = -(F^i_{\mathrm{a}} - F^i_{\mathrm{r}})$. According to HN the retarded field is attenuated; hence, $\Sigma_{j\neq i} F^j_{\mathrm{r}}$ is small compared with $\Sigma_{j\neq i} F^j_{\mathrm{a}}$. Both these arguments appear to be unacceptable—the first because the assumption of random motion, when signals can be propagated along the past null cone, appears to be unrealistic, and the second because it involves an application of the refractive index only to response fields.

Moreover, it is easily seen that these arguments make sense only after the world lines have been prescribed according to the principle of retarded causality. Thus, the question arises: 'Why should the world lines be determined in this manner?' Since the action (1.7) determines both the fields as well as the world lines, it has to be shown that the world lines obtained by using purely retarded fields are identical with the world lines given by the action principle $\delta J = 0$. In particular, given that $\ddot{z}_i(\tau_{i0}) \neq 0$, we have to show that the solution to the constrained problem

$$\delta J = 0 \qquad \ddot{z}_j(\tau_j^-(\tau_{i0})) = 0 \qquad \ddot{z}_i(\tau_{i0}) \neq 0 \tag{2.5}$$

(where $\tau_j^-(\tau_{i0})$ corresponds to the value of τ_j at which the past null cone at $z_i(\tau_{i0})$ meets z_j) is also a solution to the 'unconstrained' problem

$$\delta J = 0 \qquad \ddot{z}_i(\tau_{i0}) \neq 0. \tag{2.6}$$

Neither WF nor HN have demonstrated this, and it is not clear how this can be possible without having all the Lagrangian multipliers equal to zero. Since the Lagrangian multiplier corresponding to a constraint can be interpreted as the sensitivity to that constraint, it would follow that the real assumption is not just that accelerations along the past null cone are zero, but that the solution to the variational problem (2.6) remains unaffected by small accelerations along the past null cone. In this context the explanation due to HN might seem more appealing; however, this explanation does not appear to be convincing to the author and some others (for instance, Davies 1978, private communication), as it allows the refractive index to distinguish between stimulus and response fields.

The other method proposed in WF was that of explicit calculation. Here the problem is considered as follows. Suppose particle i is disturbed (non-electromagnetically) and radiates the time-symmetric field

$$F^i_{\text{particle}} = \tfrac{1}{2}(F^i_{\text{r}} + F^i_{\text{a}}). \tag{2.7}$$

This field, in the course of propagation, disturbs other particles in the universe, which in turn, radiate time-symmetric fields. It is required to show, by explicit calculation, that these elementary response fields add up to produce the absorber response field

$$F^i_{\text{response}} = \tfrac{1}{2}(F^i_{\text{r}} - F^i_{\text{a}}). \tag{2.8}$$

The total field attributed to particle i is then

$$F^i_{\text{tot}} = F^i_{\text{particle}} + F^i_{\text{response}} = F^i_{\text{r}}. \tag{2.9}$$

In the calculations given in WF (derivations I–III) to calculate the absorber response the fully retarded field of the particle i is used to arrive at the expression (2.8). However, to calculate the total field attributed to particle i, the expression (2.7) is used for the field of the particle. Thus the 'cycle of reasoning' used in WF is potentially circular unless

$$F^i_{\text{r}} - \tfrac{1}{2}(F^i_{\text{r}} + F^i_{\text{a}}) = \tfrac{1}{2}(F^i_{\text{r}} - F^i_{\text{a}}) = 0, \tag{2.10}$$

i.e., unless radiative damping vanishes.

In an attempt to show that the argument is not circular, in WF derivation II, the total outgoing disturbance is denoted by (?) F^i_{r}, and equation (20) of WF reads

total disturbance diverging from source	= proper retarded field of source itself	+ field apparently diverging from source actually composed of parts converging on individual absorber particles.

(2.11)

The method used by WF is such that results for the advanced field can be obtained by replacing 'retarded' everywhere by 'advanced', and 'diverging' by 'converging'. Hence, if it is not assumed, *a priori*, that (?) = 1, we also have

total disturbance converging on source	= proper advanced field of source itself	+ response field of past absorber apparently converging on source	+ response field of future absorber apparently converging on source

(2.12)

leading to

$$(1 - ?) = (\tfrac{1}{2}) + \tfrac{1}{2}(1 - ?) + \tfrac{1}{2}(?), \tag{2.13}$$

which implies (?) = 0. Thus, the argument in WF is circular, unless radiative damping vanishes.

2.2. *The divergences of* TSE

The source of the above inconsistencies can be traced to the fact that explicit calculation, even in the two-particle case, leads to divergences, and these divergences are bound to persist in the n-particle case. Thus, consider two charged particles i and j with charges e_i and e_j and masses m_i and m_j. Suppose a disturbance acts on particle i giving it a non-relativistic acceleration

$$\dot{\boldsymbol{v}}_i(t) = \boldsymbol{A}\ \mathrm{e}^{-\mathrm{i}\omega t} \tag{2.14}$$

where $\boldsymbol{A}$ is the amplitude and ω is the periodicity of the disturbance.

As a result of this acceleration, particle i radiates the time-symmetric fields $\frac{1}{2}(F_\mathrm{r}^i + F_\mathrm{a}^i)$. The field F_r^i interacts with the particle j at a later time giving it an acceleration

$$\dot{\boldsymbol{v}}_j^\mathrm{r}(t) = \frac{1}{2}\frac{e_i e_j}{m_j}\boldsymbol{A}\ \sin(\dot{\boldsymbol{v}}_i, \boldsymbol{R}_\mathrm{r})\ \exp[-\mathrm{i}\omega(t - R_\mathrm{r})]\boldsymbol{e} \tag{2.15}$$

where $\boldsymbol{R}_\mathrm{r}$ is the interparticle separation in the retarded case and $\boldsymbol{e}$ is the unit electric polarisation vector, the direction of which is taken to be negative if it has a positive component along $\dot{\boldsymbol{v}}_i(t)$.

The corresponding advanced field of j interacts with i *simultaneously* with the original disturbance to produce an additional acceleration

$$\Delta_\mathrm{a}^\mathrm{r}\dot{\boldsymbol{v}}_i(t) = \frac{1}{2}\frac{e_i^2 e_j^2}{m_i m_j}\sin^2(\dot{\boldsymbol{v}}_i, \boldsymbol{R}_\mathrm{r})\boldsymbol{A}\ \mathrm{e}^{-\mathrm{i}\omega t}. \tag{2.16}$$

Similarly, the advanced–retarded interaction between i and j leads to an additional acceleration

$$\Delta_\mathrm{r}^\mathrm{a}\dot{\boldsymbol{v}}_i(t) = \frac{1}{2}\frac{e_i^2 e_j^2}{m_i m_j}\sin^2(\dot{\boldsymbol{v}}_i, \boldsymbol{R}_\mathrm{a})\boldsymbol{A}\ \mathrm{e}^{-\mathrm{i}\omega t} \tag{2.17}$$

where $\boldsymbol{R}_\mathrm{a}$ is the interparticle separation in the advanced case.

Thus, starting from the assumption that the acceleration of i at time t is $\dot{\boldsymbol{v}}_i(t)$, we have reached the conclusion that the acceleration of i at a time infinitesimally later than t is $\dot{\boldsymbol{v}}_i(t) + \Delta_\mathrm{a}^\mathrm{r}\dot{\boldsymbol{v}}_i(t) + \Delta_\mathrm{r}^\mathrm{a}\dot{\boldsymbol{v}}_i(t)$. It is useless to try to sum up all the changes arising as a result of these stimulus and response fields because, if as a result we arrive at the value $\dot{\boldsymbol{v}}_i'(t) \neq 0$ for the acceleration of i, the above reasoning goes through with $\dot{\boldsymbol{v}}_i'(t)$ in place of $\dot{\boldsymbol{v}}_i(t)$. In fact, singularities are present at all points along the world lines of i and j where the interaction actually takes place. Thus, as a result of the interaction between i and j, the field at any point of space–time due to an arbitrary (non-zero) initial acceleration of i is indeterminate.

Schulman (1974) encountered a similar problem with respect to the differential equation

$$\ddot{x}(t) + \omega^2 x(t) = \tfrac{1}{2}\alpha x(t - \tau) + \tfrac{1}{2}\beta x(t + \sigma) + \phi(t) \tag{2.18}$$

(where α, β, τ, σ are constants, $\tau, \sigma > 0$, and $\phi(t)$ is a given function), and has suggested the use of boundary conditions to make such an equation tractable. In fact, the initial-value problem for such equations remains unsolved. Schulman has also suggested that a difference in the values of τ and σ could, perhaps, account for the suppression of advanced interactions.

Wheeler and Feynman (1949) have attempted to resolve this 'paradox' by saying that impulsive forces do not exist in nature. However, if we were to accept this point of view then the action (1.7) must be abandoned, because it assures the existence of just such impulsive forces. In this paper a somewhat similar point of view is adopted, and this is considered in greater detail below. In particular, time-symmetric fields will be dealt with, without reference to any action principle.

3. Signal velocity for advanced radiation

The roots of the above paradox lie in the assumption that a charged particle is a point charge which responds instantaneously to any incident radiation. Now electromagnetic, gravitational and quantum-mechanical considerations (Dirac 1938, 1962a, b, Raju 1979) indicate that the charges we consider must be distributed over a finite region. However, considerations of finiteness alone are not sufficient to remove the ambiguities noted above. The hypothesis of extended charges implies that the velocity with which the actual interaction between two charged particles takes place (signal velocity) is not the same as the wave velocity. Now, in the retarded case the signal velocity is lower than the wave velocity, and, as Kamat (1970) has suggested, this is true in the advanced case as well. As a result, a larger time interval is required for the actual interaction to take place in both cases. Naturally, in the advanced case this time interval is measured in the backward direction. Hence, the interaction with signal velocity takes place *earlier* than the interaction with wave velocity and not later, as suggested by Kamat (1970). By symmetry, this 'advance' in the advanced case just compensates the usual delay in the retarded case, and this brings us back to the situation in § 2.2.

However, let us consider instead a model of an extended charged particle which interacts at its boundary. Shell-like models of this type have been proposed by Dirac (1962a) and Raju (1979). The boundary of the particle in these models may be considered to be rigid in the sense that spherical symmetry is maintained, or in the sense (of Dirac 1962b) that signals can travel instantaneously in the interior of the particle. In this situation the time interval for preliminary interaction in the advanced case is fractionally longer than the corresponding time interval in the retarded case (see figure 1). It follows that, in all cases, the time interval for an actual interaction is longer in the advanced case than in the retarded case. There is, therefore, a systematic bias ensuring that the signal velocity for advanced interaction is smaller than the signal velocity for retarded interaction, i.e., the time interval in which the actual advanced interaction

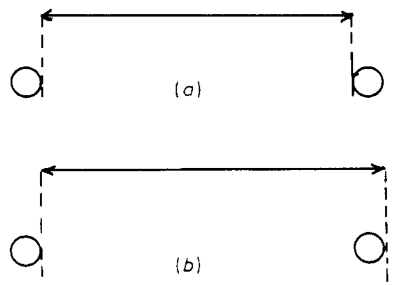

Figure 1. Difference in the time taken for preliminary interaction: (*a*) retarded interaction; (*b*) advanced interaction.

takes place is longer than the time interval in which the actual retarded interaction takes place. The relevant interaction diagram would be as in figure 2 and not as in figure 1(*b*) of Schulman (1974).

Thus the notion of an extended charged particle as a (rigid) shell leads to a natural physical justification of the hypothesis of lowered signal velocity for advanced radiation. The notion of an extended particle was also used by Dirac (1938) to account for pre-acceleration. However, Dirac's explanation of pre-acceleration is not fully satisfactory because, for the one-dimensional equations of motion,

$$m(\dot{w}-\alpha\ddot{w})=f(\tau) \tag{3.1}$$

(where m is the mass, $\alpha=\frac{2}{3}e^2/m$, e is the charge, $w(\tau)=\sinh^{-1}(\dot{z}(\tau))$, $z(\tau)$ is the world line of the particle and $f(\tau)$ is the force due to the external field), Dirac (1938) proposed the special value of the initial acceleration, given by

$$\dot{w}(0)=\frac{1}{m}\int_0^\infty e^{-\tau/\alpha}f(\tau)\,d\tau. \tag{3.2}$$

It is clear that in response to an impulse at $\tau=0$,

$$f(\tau)=K\delta(\tau), \tag{3.3}$$

the particle acceleration, given by

$$\begin{aligned}\dot{w}(\tau)&=\frac{1}{\alpha}K\,e^{\tau/\alpha} &\qquad \tau<0\\ &=0 &\qquad \tau>0,\end{aligned} \tag{3.4}$$

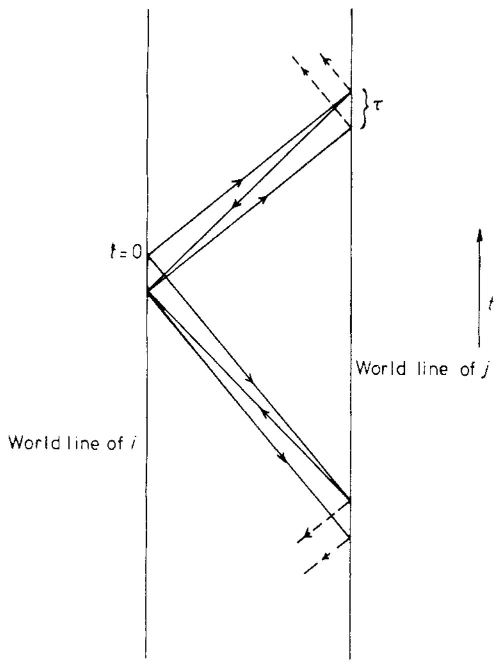

Figure 2. Interaction diagram for two particles, illustrating the effect of lowered signal velocity for advanced radiation. The time difference, τ, is grossly exaggerated.

is non-zero (though small) for large negative values of τ. Dirac (1938) attempted to explain this by saying that the classical electron has no boundaries and can, therefore, 'feel' the impulse at large times before it is actually applied. This explanation no longer appears reasonable in the context of a definite model for the electron that has a definite size or some size smaller than a definite size.

There is an alternative explanation in the present framework, because the charged particle radiates and interacts with other charged particles if an impulse is applied at $\tau = 0$, say. Due to the lowered signal velocity of advanced radiation, the effects of this impulse are propagated (in a rapidly decaying manner, if retarded radiation is approximately consistent) along the entire portion of the world line for $\tau \leqslant 0$. Thus it is not the indefinite size of the electron but the lowered signal velocity for advanced radiation which accounts for pre-acceleration at large times before an impulse is applied.

Finally, we note that there is no serious disadvantage associated with the fact that the hypothesis we are making apparently does not have an immediate generalisation to the quantum-mechanical case (see, however, Raju 1979). This is true, if only because any really successful theory of quantum mechanics will incorporate relativistic considerations, and be able to treat particles with an extended structure.

4. The absorber response

We consider only the case of an isolated charged particle which is disturbed by some non-electromagnetic force and radiates time-symmetric fields. As pointed out by Hogarth (1962), Sciama (1963) and Hoyle and Narlikar (1964), the effective interaction with the absorber takes place across cosmological distances. At these distances, the part of the universe (the past absorber) which interacts with the advanced component of the field of the charged particle must be considered to be physically distinct from the part (the future absorber) which interacts with the retarded component. The situation is further simplified by the fact that the charged particle can receive only retarded radiation from the past absorber, and only advanced radiation from the future absorber.

It will be understood in the following that we are dealing with plane-polarised fields of a fixed frequency ω. Since the theory is linear and since the results are indepedent of ω, the results hold by Fourier superposition for more complicated fields. Following figure 3, we let S_i^{P} and S_i^{F} be the various stimulus fields for the past absorber and the future absorber, while R_i^{P} and R_i^{F} denote the corresponding response fields. Because the difference in time intervals for retarded and advanced interactions depends only on the (average) size of the charged particles, and not on the interparticle separation, all the absorber particles can be lumped together in the interaction diagram. However, the multiparticle nature of real absorbers might affect another assumption, that of linearity, which, in this context, refers to spherical symmetry (Hogarth 1962). Such possible departures from linearity will not be considered here and we assume that all the response fields are linearly related to the stimulus fields, the response factors being given by p and f for the past and future absorbers respectively.

Denoting the advanced and retarded source fields by F_{a} and F_{r}, we have

$$\begin{aligned} S_1^{\mathrm{P}} &= \tfrac{1}{2}F_{\mathrm{a}} = \tfrac{1}{2}(F_{\mathrm{a}} - F_{\mathrm{r}}) \qquad & S_1^{\mathrm{F}} &= \tfrac{1}{2}F_{\mathrm{r}} = \tfrac{1}{2}(F_{\mathrm{r}} - F_{\mathrm{a}}) \\ R_1^{\mathrm{P}} &= \tfrac{1}{2}p(F_{\mathrm{r}} - F_{\mathrm{a}}) \qquad & R_1^{\mathrm{F}} &= \tfrac{1}{2}f(F_{\mathrm{a}} - F_{\mathrm{r}}) \end{aligned} \tag{4.1}$$

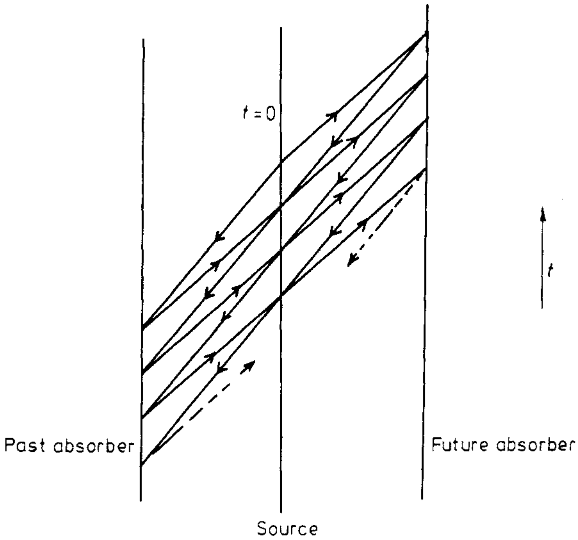

Figure 3. Interaction diagram for an 'isolated' particle.

where the first line of (4.1) is obtained by noting that the field F_r is zero in the region of the past absorber and F_a is zero in the region of the future absorber. In the last line of (4.1) we have temporarily discarded any possible difference between a converging retarded field and an advanced field. However, this does not require any new assumption since we have already assumed the particle to be isolated.

The other stimulus and response fields are given by

$$\begin{array}{lll} S_2^{\mathrm{P}} \doteq R_1^{\mathrm{F}} & S_2^{\mathrm{F}} \doteq R_1^{\mathrm{P}} & R_i^{\mathrm{P}} \doteq p S_i^{\mathrm{P}} \\ \vdots & \vdots & \vdots \\ S_n^{\mathrm{P}} \doteq R_{n-1}^{\mathrm{F}} & S_n^{\mathrm{F}} \doteq R_{n-1}^{\mathrm{F}} & R_i^{\mathrm{F}} \doteq f S_i^{\mathrm{F}}. \end{array} \tag{4.2}$$

We now assume that, from the point of view of macroscopic observation, all the response fields act simultaneously with the initial source fields. As a particular consequence of this assumption, the resulting electromagnetic arrow of time, like the thermodynamic arrow of time, will be applicable only at a macroscopic level.

With the above assumption, the total field

$$F_{\mathrm{tot}} = \tfrac{1}{2}F_r + \tfrac{1}{2}F_a + \sum_{i=1}^{\infty} R_i^{\mathrm{P}} + \sum_{i=1}^{\infty} R_i^{\mathrm{F}}. \tag{4.3}$$

Now, ignoring those terms which cancel with $\Sigma_{i=1}^{\infty} R_i^{\mathrm{F}}$, and assuming $|pf| \neq 1$,

$$\begin{aligned} \sum_{i=1}^{\infty} R_i^{\mathrm{P}} &= [p + pfp + p(fp)^2 + \ldots]\tfrac{1}{2}(F_r - F_a) \\ &= \left(p \sum_{n=0}^{\infty} (pf)^n\right)\tfrac{1}{2}(F_r - F_a) \\ &= \left(\frac{p}{1-pf}\right)\tfrac{1}{2}(F_r - F_a). \end{aligned} \tag{4.4}$$

Similarly,

$$\sum_{i=1}^{\infty} R_i^{\mathrm{F}} = \left(\frac{f}{1-pf}\right)\tfrac{1}{2}(F_{\mathrm{a}}-F_{\mathrm{r}}). \tag{4.5}$$

From (4.4) and (4.5)

$$F_{\mathrm{tot}} = \frac{1}{2}\left(1+\frac{p}{1-pf}-\frac{f}{1-pf}\right)F_{\mathrm{r}} + \frac{1}{2}\left(1+\frac{f}{1-pf}-\frac{p}{1-pf}\right)F_{\mathrm{a}}. \tag{4.6}$$

Hence the condition for $F_{\mathrm{tot}} = F_{\mathrm{r}}$ is

$$(p-f)/(1-pf) = 1 \tag{4.7}$$

or

$$p(1+f) = (1+f). \tag{4.8}$$

Equation (4.8) is valid if either $p = 1$ with f arbitrary, or $f = -1$ regardless of the value of p, given that $|pf| \neq 1$. Similarly, we have for the existence of purely advanced radiation the conditions $p = -1$ or $f = 1$, $|pf| \neq 1$.

If we know *a priori* that $p = f = 1$ then (4.7) is indeterminate. However, in this case $R_n^{\mathrm{P}} + R_n^{\mathrm{F}} = 0$ for each n, i.e., the response fields cancel termwise. Hence, in the expression (4.3) for F_{tot}, we are left only with the original time-symmetric fields of the particle. Thus, in the case $p = f = 1$ the nature of the radiation is time symmetric. On the other hand, if we only have $p \simeq 1$, $f \simeq 1$ then a mixture of the form $(1-\delta)F_{\mathrm{r}} + \delta F_{\mathrm{a}}$ can exist, with

$$\delta = \frac{(1-p)-\frac{1}{2}(1-p)(1-f)}{(1-p)+(1-f)-(1-p)(1-f)} \simeq \frac{(1-p)}{(1-f)} \ll 1 \tag{4.9}$$

provided $(1-p) \ll (1-f)$†.

Strictly speaking, since any measurement will involve only a finite portion of the world line of the particle, the observed radiation will always be a mixture, and the empirical testability of this prediction is considered in § 6.

5. The absorber mechanism

It is now necessary to calculate the values of p and f for various cosmological models. The main interest, of course, lies in finding those cases for which $p = 1$, $f \neq 1$. We first consider qualitatively the mechanism underlying the absorber response. This has never, in any case, been clearly brought out. Thus, for instance, Kamat (1970) questions the assumption that this response is emitted in a 'backwards' direction, and feels that the existence of a 'backwards refractive index' is necessary for the existence of such a response. On the other hand, Pegg (1975b) has maintained that the refractive index arises after consideration of the absorber response, and has quoted the derivations in WF in support of his contention. However, in these derivations the interaction of the advanced component of the elementary response fields with the charged particles in the absorber has not been considered at all, implying that a special choice for the refractive index for advanced radiation has already been made. As such, the origin of the backwards refractive index remains a mystery.

† This is *not* the same as saying $f \ll p$. For instance, if $p = 1-10^{-20}$ and $f = 1-10^{-10}$, then $(1-p) \ll (1-f)$ is true, but not $f \ll p$, in the usual sense.

This mystery is, however, easily resolved by considering the phase relations for the elementary response fields. Consider a retarded stimulus field, incident on an absorber which we assume to be a dilute plasma of charged particles. For the purpose of understanding the origin of the backwards response field it is sufficient to consider the case of a plane monochromatic field, which we denote at a plane z in the absorber by $F_r = F_0 e^{i\omega t}$. The phase of the field at the position $z + \delta z_i$ of particles i (see figure 4) is $\omega(t - \delta z_i)$. We assume that the damping is very light and the refractive index is approximately unity, so that the phase of the re-emitted retarded field of particle i at $z + \delta z$ in the direction θ is

$$\omega(t - \delta z_i) - \omega[(\delta z - \delta z_i)/\cos\theta].$$

Thus in the forward direction ($\theta \approx 0$) the re-emitted retarded fields interfere constructively and are in phase with the original retarded field. In any other direction the interference is destructive because of the random positioning of the particles.

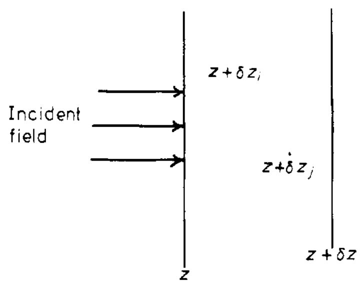

Figure 4. Origin of the backward response field: the re-emitted advanced fields interfere constructively in the backward direction, and destructively elsewhere.

But, for the advanced field the phase at z is $\omega(t - \delta z_i) + \omega\delta z_i \pm \pi$, whereas the phase at $z + \delta z$ is $\omega(t - \delta z_i) - \omega[(\delta z + \delta z_i)/\cos\theta]$. It follows that the re-emitted advanced fields interfere constructively in the backwards direction and destructively elsewhere. Moreover, they are exactly π out of phase with the incident retarded field. The additional phase difference of π appears because the re-emitted retarded fields are in phase with the incident field whereas the re-emitted advanced fields are π out of phase with the re-emitted retarded fields.

This explains the origin of the backward response field, but now there is a new mechanism of absorption at work in the time-symmetry normalisation factor $\frac{1}{2}$. Assuming an extinction theorem of the Ewald–Oseen type (see, for instance, Born and Wolf 1964), only half of the absorbed energy is re-emitted in the forward direction, while the other half is converted into the energy of the absorber response. Thus, it is time-symmetric scattering rather than thermal absorption which gives rise to the absorber response. Thermal absorption, if it occurs, will permanently remove the energy available for the absorber response. It should be observed that secondary and tertiary scattering also contribute to the absorber response, because in each case the response field exactly retraces the path of the incident stimulus field. In actuality, an infinity of interactions might take place within the absorber, but we do not have to consider this, since we can get the magnitude of the response field by straightforward energy considerations. Thus, if there are sufficiently many charged particles in the absorber so that the incident field loses all its energy (by scattering) then all this lost energy must appear in the form of the absorber response, i.e., the absorber is ideal. In this case any significant motion of the charged particles in the absorber is confined to very small time intervals and consequently there is very little thermal absorption.

Applying these considerations to the past and future absorbers we see that $0 \leq p, f \leq 1$ and $p = 1$ ($f = 1$) provided the past (future) absorber has a sufficiently large number of charged particles.

This brings us to the problem of determining those cosmological models that satisfy the conditions for the existence of retarded radiation. Here we will consider the situation only with respect to evolutionary models. The past null cone in this case is opaque ($p \simeq 1$) due to the initial singularity. Along the future null cone we assume that absorption occurs mainly by discrete objects, which absorb their geometric cross section of the incident flux. In that case, models which expand with $R \propto t^{2/5}$ or more are transparent along the future null cone (Davies 1972b). These models, which include the Einstein–de Sitter model, therefore satisfy the requirements for the existence of retarded radiation.

The situation is not so clear with regard to models with a final singularity, because, as seen in § 3, retarded radiation can be consistent even if $p \simeq 1$ and $f \simeq 1$. In particular, the possibility raised by Gold (1967) is not ruled out. For a source at large times from the initial singularity, some of the emitted radiation would undergo large blue shifts and correspondingly greater losses due to (nonlinear) thermal absorption and pair production. Hence $(1-p)$ would increase as the source moves away from the initial singularity. In this manner it is possible that p ultimately falls below the value of f. Similarly, due to the peculiarities of absorption in an epoch dominated by advanced radiation, $(1-f)$ could decrease towards the final singularity.

Thus there is a good possibility that, at least in this case, the cosmological arrow of time determines the electromagnetic arrow of time, although a deeper investigation would be required before drawing any firm conclusion.

6. Empirical detection of advanced radiation

In § 4, the conditions for the existence of purely retarded radiation were derived under the assumption that all the stimulus and response fields act simultaneously at any point in space. In actuality, according to the basic hypothesis, the nth response field acts τ seconds earlier than the $(n-1)$th response field, where τ is the characteristic delay associated with the signal velocity of advanced radiation. The precise value of τ would depend on the particular model under consideration. However, for any realistic model of a particle of finite size, τ would be very small. In fact, if the model proposed by Dirac (1962*a*, *b*) and Raju (1979) is used, the value of τ would definitely be smaller than 10^{-23} s. A macroscopic observer would carry out an observation only in a finite time interval, T, large compared with τ. A large number, $N = (T/\tau)$, of the response fields would act within this time interval, and so the assumption of simultaneity would be justified *provided* the series converges with sufficient rapidity. Since retarded radiation is approximately consistent, i.e., $p \simeq 1$, the series would converge rapidly, unless $f \to 1$. But, by using increasingly efficient local absorbers, in theory it can always be arranged to have $f \to 1$ in the laboratory. Hence, if the theory is correct, advanced radiation can be detected experimentally.

Since one experiment of this nature has already been carried out (Partridge 1973) and another proposed (Heron and Pegg 1974), it would be worthwhile discussing these theoretical predictions in this context. Partridge's experiment consisted of measuring the power input to a horn antenna as it radiated alternately into free space and a local absorber. Partridge assumed a relationship of the type

$$P_{\mathrm{f}} = (1-\delta)P_{\mathrm{a}} \tag{6.1}$$

where P_{f} and P_{a} denote the power inputs while radiating into free space and the local absorber respectively. According to the theories in WF and HN, $\delta \geqslant 0$. According to the present theory, the presence of a local absorber would increase the content of advanced radiation in the mixture, and hence δ should be negative. The mean value of δ obtained by Partridge was $(-1{\cdot}1 \pm 1{\cdot}6) \times 10^{-9}$, and Partridge concluded that this was not significant. However, in obtaining this mean value Partridge took a weighted average over various phase settings of a phase-sensitive detector, and, for the two phase settings $\phi = 0°$ and $\phi = 180°$ for which the detector is most sensitive, the values of δ obtained were significant and negative. Although the possibility of instrumental error can by no means be ruled out, these results are certainly suggestive, and the experiment deserves to be repeated with greater sensitivity.

In the proposed experiment by Heron and Pegg, a 'dynamic' local absorber is to be used, which, it is supposed, would affect only the value of p or that of f. If only the value of p is sought to be increased, then if $p = 1$ there would be no change in the power input, whereas if $p \simeq 1$, $p \neq 1$, then the power input would increase.

7. Conclusions

Retarded radiation is consistent in those models which satisfy $p = 1$ or $f = -1$, with $|pf| \neq 1$. The Einstein–de Sitter model satisfies these conditions approximately. In the closed Friedman model it is likely that retarded radiation is dominant during expansion and advanced radiation during contraction.

The theory predicts that advanced radiation exists in small amounts, and can be detected experimentally.

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